Centre Number

Student Number

Plume Specialist Tuition

2026 **HIGHER SCHOOL CERTIFICATE PRACTICE EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–13)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 The value of

$$\tan^{-1}\left(\tan \frac{5\pi}{6}\right)$$

is

A. $-\frac{\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{5\pi}{6}$

D. $-\frac{5\pi}{6}$

2 The polynomial

$$P(x) = x^4 - 2x^3 + ax^2 + bx + 12$$

is divisible by $(x-2)(x+1)$. What is the value of b?

A. -8

B. -2

C. 2

D. 8

3 Let

$$f(x) = \sqrt{x-1} \qquad \text{and} \qquad g(x) = \frac{1}{x-2}.$$

What is the domain of the composite function $f \circ g$?

A. $(-\infty, 2)$

B. $(2, 3]$

C. $[3, \infty)$

D. $(-\infty, 2) \cup [3, \infty)$

4 What is the coefficient of x^5 in the expansion of

$$(1+2x)^6(1-x)^4\,?$$

A. -72

B. -36

C. 36

D. 72

5 Let $\mathbf{a} = (3, 1)$ and $\mathbf{b} = (1, 2)$. The magnitude of $\mathbf{a} + \lambda\mathbf{b}$ is minimised when

A. $\lambda = -2$

B. $\lambda = -1$

C. $\lambda = 1$

D. $\lambda = 2$

6 A random variable X has distribution $\mathrm{Bin}(n, p)$, where

$$\mathrm{E}(X) = 6 \qquad \text{and} \qquad \mathrm{Var}(X) = \frac{9}{2}.$$

What is the value of n?

A. 8

B. 12

C. 18

D. 24

7 How many solutions does

$$\cos 2x = \sin x$$

have for $0 \leq x \leq 2\pi$?

A. 2

B. 3

C. 4

D. 5

8 The value of

$$\int_0^1 \frac{1}{(1+x)\sqrt{x}}\,dx$$

is

A. $\dfrac{\pi}{4}$

B. $\dfrac{\pi}{2}$

C. $\ln 2$

D. $2\ln 2$

9 Newton's method is applied to $f(x) = x^3 - x - 1$ with first approximation $x_1 = 1$. What is the third approximation x_3?

A. $\frac{23}{16}$

B. $\frac{31}{23}$

C. $\frac{46}{31}$

D. $\frac{3}{2}$

10 For $a > 0$, the tangent to the curve $y = \ln x$ at $x = a$ passes through the origin. What is the value of a?

A. 1

B. $e^{1/2}$

C. e

D. e^2

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) The monic cubic polynomial **2**

$$P(x) = x^3 + px^2 + qx + 6$$

has roots α, β and γ, where

$$\alpha + \beta = 1 \qquad \text{and} \qquad \alpha\beta = -2.$$

Find p and q.

(b) Solve, for $x > 0$, **2**

$$\tan^{-1} x + \tan^{-1}(2x) = \frac{\pi}{4}.$$

Give your answer in exact form.

(c) A committee of five is chosen from six women and five men. The committee must contain at least two women and at least two men. One particular woman and one particular man refuse to serve together on the committee. **3**
How many different committees can be formed?

(d) A random variable X has a binomial distribution. It is known that **3**

$$\Pr(X = 1) = 5\Pr(X = 0) \qquad \text{and} \qquad \Pr(X = 2) = 2\Pr(X = 1).$$

Determine the parameters n and p, and hence find $\Pr(X \geq 4)$.

Question 11 continues on page 8

Question 11 (continued)

(e) Evaluate **2**

$$\int_0^1 \frac{x}{1+x^4}\,dx.$$

(f) Solve **3**

$$2\sin x + 2\sin 3x = \cos x$$

for $0 \le x \le 2\pi$. Give exact answers, using inverse trigonometric notation where necessary.

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet

(a) Points A, B and C have coordinates **3**

$$A = (1,2), \qquad B = (5,-1), \qquad C = (k,4).$$

If $\angle BAC = 45°$, find k.

(b) Use mathematical induction to prove that **3**

$$48 \mid \left(7^{2n} - 1\right)$$

for every positive integer n.

(c) A point $P = (x, y)$ moves along the branch $xy = 12$ in the first quadrant. At an instant when $x = 3$, the coordinate x is increasing at 2 units per second. **3**
Find the rate of change of the distance OP at this instant, where O is the origin.

(d) How many strings containing exactly five letters A and three letters B can be formed if no two B's are adjacent? **2**

(e) For $a > 0$, the tangent to the curve $y = e^{-x}$ at the point where $x = a$ meets the coordinate axes at P and Q. **4**
Find the value of a for which the area of triangle OPQ is greatest, and find this greatest area.

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) Let **a** and **b** be vectors satisfying

$$|\mathbf{a}| = 4, \qquad |\mathbf{b}| = 6, \qquad \mathbf{a} \cdot \mathbf{b} = 12.$$

Points A and B have position vectors **a** and **b**, respectively. A point P lies on the line segment AB and has position vector

$$\mathbf{p} = (1 - t)\mathbf{a} + t\mathbf{b}, \qquad 0 \le t \le 1.$$

(i) Find $\angle AOB$, where O is the origin. **1**

(ii) Show that **2**

$$OP^2 = 16 - 8t + 28t^2.$$

(iii) Find the value of t for which OP is minimised, and find the minimum value of OP. **2**

(iv) Show that, at this value of t, OP is perpendicular to AB. **2**

Question 13 continues on page 11

Question 13 (continued)

(b) Define

$$f(x) = \tan x - 2x, \qquad 0 < x < \frac{\pi}{2}.$$

The equation $f(x) = 0$ has a solution α in the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

(i) Show that α, the solution of $f(x) = 0$ in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, is unique. **3**

(ii) Starting with $x_1 = 1.2$, use one iteration of Newton's method to obtain an improved approximation to α. Give your answer correct to three decimal places. **2**

(iii) Show that **1**

$$\sin 2\alpha = \frac{4\alpha}{1 + 4\alpha^2}.$$

(iv) Hence, or otherwise, evaluate **2**

$$\int_0^{\alpha} \frac{1}{1 + 4x^2}\, dx$$

in terms of α.

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Let α, β and γ be the roots of

$$x^3 - 3x + 1 = 0,$$

and define

$$S_n = \alpha^n + \beta^n + \gamma^n \qquad (n \geq 0).$$

(i) Show that the equation $x^3 - 3x + 1 = 0$ has three distinct real roots. **2**

(ii) Find S_1 and S_2. **2**

(iii) Show that, for $n \geq 3$, **2**

$$S_n = 3S_{n-2} - S_{n-3}.$$

(iv) Hence find S_6. **2**

(v) Find **1**

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}.$$

Question 14 continues on page 13

Question 14 (continued)

(b) The word MATHEMATICS contains 11 letters, with M, A and T each occurring twice.

(i) Find the total number of distinct arrangements of MATHEMATICS. **1**

(ii) Find the number of distinct arrangements in which no two identical letters are adjacent. **4**

(iii) If an arrangement is chosen uniformly at random, find the probability that no two identical letters are adjacent. **1**

End of paper

BLANK PAGE

BLANK PAGE

BLANK PAGE

Plume.

Plume Specialist Tuition

2026 HIGHER SCHOOL CERTIFICATE PRACTICE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

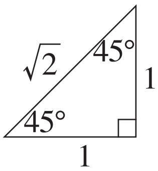
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

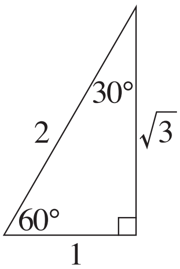
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

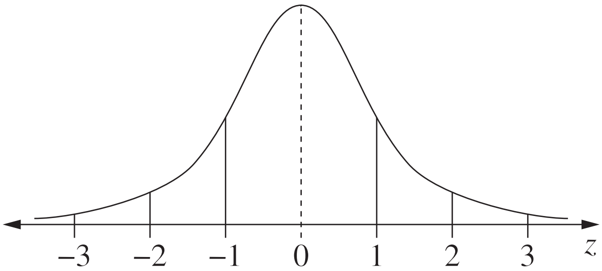
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x-\mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\big[f(x)\big]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \big[f(x)\big]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \big[f(x)\big]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \big[f(x)\big]^2}$

Integral Calculus

$$\int f'(x)\big[f(x)\big]^n dx = \frac{1}{n+1}\big[f(x)\big]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\big|f(x)\big| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \big[f(x)\big]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \big[f(x)\big]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\Big\{f(a) + f(b) + 2\Big[f\big(x_1\big) + \cdots + f\big(x_{n-1}\big)\Big]\Big\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$